ComputerModeling of Phenomena in Dynamical Systems
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Компьютерное моделирование явлений в динамических системах
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***Absract**: earlier, author [4] proposed systematical search of effects and phenomena in mathematics. Some mathematical results are too complicated to be proven deductively but can be corroborated by computational experiments. There are presented in the paper:*
- the practical machine splitting phenomenon (for computer programs transforming the set of machine numbers into itself);
- (a common Kyrgyz word) $irgöö$ means: discrete optimization by means of synergetic, or the phenomenon «random vibration of balls of different sizes of same material in a wide vessel yields migration of the biggest one to the center of their surface»;
- phenomenon of transforming of smooth solution of degenerate equation of a singularly perturbed differential one: while the small parameter vanishes, the solution tends to another continuous function.
***Аннотация:** на основе введенных рамочных определений понятий эффекта и явления в статье представлены:*
- явление, соответствующее экспериментально установленному явлению, обозначаемому словом «иргөө» (дискретная оптимизация посредством синергетики);
- явление машинного практического расщепления;
- явление преобразования решения вырожденного уравнения сингулярно-возмущенного дифференциального уравнения. Все представленные примеры будут способствовать развитию теории динамических систем и расширению ее приложений.

***Keywords**: phenomena in mathematics, computational experiment, singular perturbation, differential equation.*
***Ключевые слова:** явления в математике**,** вычислительный эксперимент, сингулярное возмущение, дифференциальное уравнение.*

Introduction

Discoveries of new «phenomena» and «effects» used to be sufficient steps in developing science but there were not definitions of these notions before our publication [2]. We gave corresponding definitions and examples, proposed methodic to search new phenomena.

Considered a theorem in general as an implication of conditions $A \Rightarrow B$, or, more concrete, if there is a general class X of objects x and $A \subset X, B \subset X$ then $A \subset B$ or $(x \in A) \Rightarrow (x \in B)$. To search «phenomena» and «effects» more systematically we proposed.

Definition 1 (generalizes the approach of the catastrophe theory). To prove sufficiency of A for B one is to construct an example without both A and B. An (interesting, nonpresumable, single) way of violating B is said to be a *phenomenon*. The notion «single» can be defined more exactly. Let X be a set and a measure can be introduced in it. Then a subset $P \subset X \backslash B$ is a phenomenon if $mes(P) = 0$. In other words, if $x \in X$ then $x \in P$ «almost never».

Definition 2. If P is a property (or some properties) of elements $x \in X$ possessing a property E such that a logical proof $(E \wedge C) \Rightarrow P$ (where C is any additional condition) is too complicated and the property P was discovered not by a logical way but by meeting paradoxes, by experiments in physics and chemistry or by computational experiments in mathematics then E is said to be an *effect*.

These definitions yield the following methodic. If some objects $x \in X$ with different but similar unexpected properties have the same property E then this property is considered to be an effect. Putting additional conditions on x new phenomena may be found in the class X.

Various Examples of Well-known Phenomena

To demonstrate definitions given above we mention the following from our viewpoint.

Example 1. The phenomenon of equivalence of an infinite set to its proper subset (Galileo) as a consequence of effect of infinity; the set theory (G.Cantor) had been developed on this base.

Example 2. The phenomenon of one-sidedness of a surface (Moebius band and Klein bottle) caused creation of combinatorial topology.

Example 3. The phenomenon of bifurcation (non-uniqueness of solution of an initial value problem for a dynamical system); the catastrophe theory had been developed while investigating this phenomenon.

Example 4. As the display is real, the following hypothesis was stated [1]. The display presents to the human the new kind of motion (except mechanical, physical, chemical, biological etc.). By this hypothesis, the following phenomenon of «non-returning» as the consequence of the effect of «non-Euclideanism» was constructed [2]. If the user moving in a kinematical space led by the common intuition tries to return to the same place then s/he finds her/himself among transformed environment (Moebius band) or in another place (Riemann surfaces).

Let T and Y be topological spaces, ε be a small positive parameter, ε be an upper boundary for it and $\{y_\varepsilon(t): T \to Y\}$ be a family of continuous functions.

Example 5. The phenomenon of boundary layer in physics of liquids and gases (zero-velocity of the layer of liquid or gas contiguous to a solid (L. Prandtl)); the theory of singular perturbations had been developed on this base. This phenomenon is well-known and there were physical definitions. for instance «the portion of fluid adjacent to the surface of an object around which the fluid is flowing» but we did not find any strict mathematical definition for it. For T and Y being metrical spaces we proposed the following.

Definition 3. If

$$(\exists t_0 \in T)(\forall a > 0)(\exists \varepsilon_0)(\forall \varepsilon < \varepsilon_0)(\exists t_1 \in T)$$
$$((\rho_T(t_0, t_1) < \varepsilon) \wedge (\rho_Y(y_\varepsilon(t_0), y_\varepsilon(t_1)) > a))$$

then a fixed boundary layer occurs.

Similar definitions can be given for other types of boundary layers: moving, receding, deepening.

Practical Machine Splitting Phenomenon

Consider any computer program P implementing any transformation of the set M of computer numbers into itself.

Definition 4 [1]. If there exists such segment $D = [D_-, D_+] \subset M$ that $(\exists x \in M)(P(x) < D_-)$, $(\exists x \in M)(P(x) > D_+)$ and $P(x_0) \in [D_-, D_+]$ for no more than one $x_0 \in M$ then the practical machine splitting phenomenon takes place.

Example 6. Approximate solving an initial value problem for a stiff ordinary differential equation.

$$\varepsilon y'(t) = (1 - y^2(t))y(t),\ 0 \le t \le 1, \varepsilon = 0.01, \quad (1)$$
$$y(0) = y_0. \quad (2)$$

Applying Euler method, step $= 0.01$.

Computer program in pascal

program split; uses crt; var i: integer; y,y0,y1: double; begin clrscr; writeln(' Input initial value y(0)'); readln(y0); y: = y0; for i: = 1 to 100 do begin y: = y+0.01*100.*(1.-y*y)*y; end; y1: = y; writeln(' y(1)~',y1:18:10); readln; end.

This program transforms y_0 to y_1.

We may let $D := [-0.5, 0.5]$, only $y_0 = 0$ transforms into D.

Synergetic Phenomenon for System of Random Difference Equations

Definition 5. (A common Kyrgyz word) $irg\ddot{o}\ddot{o}$ means: discrete optimization by means of synergetic, or the phenomenon «random vibration of hard balls of different sizes and of same material in a wide vessel yields migration of the biggest one to the center of their surface».

This experimental fact is too difficult to be proven by any mathematical model but is validated by numerical experiments with a system of difference equations.

Thus, we stated.

Hypothesis [6]. For a large number of balls in a vessel, in a certain class of processes described by random difference equations, the probability of the event «the biggest ball is close to the center of surface of heap of balls» as time tends to infinity is 1.

The cylinder of radius 1 is taken as a vessel.

Let a (large) natural number n and (small) positive radii $r_1 > r_2 \geq \ldots \geq r_n$ be given.

Definition 6. If a set of n points $\{(x_k, y_k, z_k): k = 1..n\} \subset R^3$ meets the conditions

1) $(\forall k \in 1..n)((r_k \leq z_k) \wedge (x_k^2 + y_k^2 \leq (1 - r_k)^2)$

(all balls are in the vessel);

2) $(\forall k \neq j \in 1..n)((x_k - x_j)^2 + (y_k - y_j)^2 + (z_k - z_j)^2 \geq (r_j + r_k)^2)$

(the balls do not overlap)

then such set is said to be *admissible*.

Definition 7. A (short) vector $\{u, v, w\} \in R^3$ $(w < 0)$ is said to be *admissible* for a given admissible set of points and a number $k \in 1..n$ if the set obtained by means of changing the k-th point to the point $(x_k + u, y_k + v, z_k + w)$ is admissible too.

We will call such passing from one set of points to a new set of points an *admissible* shift.

Algorithm. For any initial admissible set of points, repeat the following steps:

1) shift all points up with a (short) vector;

2) while it is possible, execute random admissible shifts (down) in the obtained admissible set of points.

The adjusted hypothesis is the following. With the probability 1, there exists such number M that after M steps there will be $x_1^2 + y_1^2 \leq r_1^2$ and there will not be other points over this point.

To verify this hypothesis a program implementing Algorithm was written in *pascal* for $n = 50$ and $r_k = 0.3 - 0.01k, k = 1..19; r_k = 0.1, k = 20..50$. Some runs of this program gave similar results corroborating the hypothesis with $M < 100$.

Remark. The phenomenon of Benard convection cells (1900) is mentioned in literature as the first example of synergetic process in a dissipative system. As the word *irgöö* existed in the Kyrgyz language some hundred years ago, this is the earlier example of synergetic process.

Phenomenon of Partial Transformation of Solution of Degenerate Equation

Consider the differential equation

$$F(t)F'(y(t))y'(t) = F'(t)F(y(t)), t \in [-1,1], \tag{3}$$

with the initial condition

$$y(-1) = y_0 \tag{4}$$

where F is a smooth function, $sgn\ F(t) = sgn\ t$, and the perturbed one

$$(\varepsilon + F(t))F'(y_\varepsilon(t))y_{\varepsilon'}(t) = F'(t)F(y_\varepsilon(t)), t \in [-1,1]. \tag{5}$$

If F is an even function then the known phenomenon of partial rotation of solution of degenerate equation takes place. For integral equations of the third kind it was proven [5].

Definition 7. If the initial value problem (3)-(4) has a smooth solution $y_0(t)$ and for the solution of (5)-(4) $Y_0(t) = lim_{\varepsilon \to 0} y_\varepsilon(t)$ exists but for some domain $T \subset [-1,1]: |Y_0(t)| \neq |y_0(t)|$ and for $t \in [-1,1] \backslash T: Y_0(t) = y_0(t)$ then the phenomenon of partial transforming of solution of degenerate equation takes place.

To prove occurrence of this phenomenon for non-even functions is too difficult and we used the Runge-Kutta method in *MathCad* for $F(t) = t^2 + 0.2t^3, \varepsilon = 0.002, \varepsilon = 0.001, \varepsilon = 0.0002$. The results corroborated the existence of $Y_0(t)$ and fulfilling of assertions in Definition 7.

Conclusion

We hope that this paper would attract attention to systematic search of new phenomena in the theory of differential and difference equations by means of computational experiments. Also, combinations of computational and real experiments would find new phenomena caused by vibrations of large number of similar objects.

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